ACTUATOR DESIGN PROTOCOL

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Actuator Systems Report Draft - CDRL A002

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1.0 INTRODUCTION

Actuators are critical to systems that control mechanical devices. With the arrival of inexpensive digital logic, the interest in smart mechanical systems has dramatically increased. A central problem to such systems is developing actuators that have high performance, while minimizing their volume, weight, power consumption, and cost.

Actuators use a wide range of technologies, hydraulic, electrodynamic, and solid state. A problem that often confronts the designer is deciding which technology to use for a particular application. The purpose of our program is to develop an umbrella set of performance measures for comparing diverse actuator technologies using a comparison basis that is the same for all. In addition, we develop guidelines that will help systematize the design process.

This report is the second in a series. In this report, we extend the previously developed general static linear actuator model to include frequency domain analysis of linear systems and time domain analysis of nonlinear systems. A formalism is developed for selecting the proper actuator so that the force transferred to the load is maximized, given a load force spectrum, load impedance, and source impedance. Even with the proper actuator, saturation often limits the force delivered to the load. Saturation can occur in either the source or the load itself. We develop a nonlinear model for source and load saturation that is used to investigate the effects of actuator system saturation on performance. This model and the actuator model provide estimates for system power output. A procedure is developed for selecting the best combination of actuator and source technologies. It uses system power output as a performance measure.

2.0 FORCE TRANSFER OPTIMIZATION

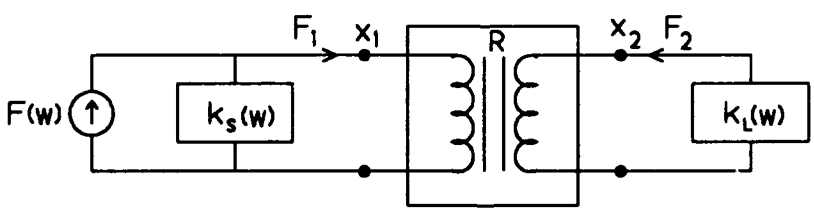
In the previous report, we developed a static model for the actuator system. In that analysis an efficient actuator system was one that maximized energy transfer to the load. Since a frequency dependent system is now being considered, a dynamic model is necessary. Energy analysis can no longer be used as a measure of system efficiency since the energy equations do not have direct equivalents in the frequency domain. For the dynamic model, optimizing actuator efficiency will be defined as maximizing the mean square force, $< F^2 >$, transferred to a fixed load impedance. This measure has the characteristic that in the static limit its optimization and the previously developed energy transfer optimization give the same result.

In developing the dynamic system model, three models for the actuator are presented. Each model expands the complexity of the needed analysis and generality of the solution. Not all applications will require the most general solution and designers will benefit from using the simpler solutions.

The first model for the actuator is an ideal transformer. The transformer lends itself to easy analysis and visualization. A second model uses the general linear element developed earlier where the actuator is represented by a static stiffness matrix. In the third actuator model we introduce a frequency dependent stiffness matrix. Frequency dependence in the actuator can arise from several sources. A common source is internal dynamics. For example, power dissipation in the coil surrounding a magnetostrictive actuator or the mass of the moving coil in a moving coil actuator.

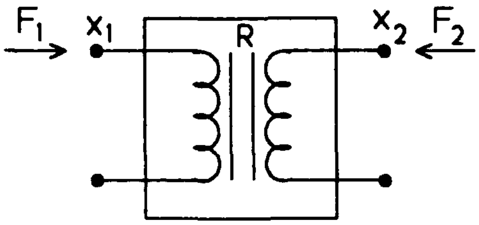
2.1 IDEAL TRANSFORMER

In its simplest form, the dynamic actuator system consists of an ideal transformer coupling a frequency dependent load and source.



(F)

This transformer is ideal. Physical transformer input and output variables can be either electrical or mechanical. We will use the notation of mechanical variables i.e., stiffness. The four terminal network for the transformer is shown below.



(F2)

The input and output variables are related by the transformer ratio, R, and this relation defines the ideal transformer.

$$F_1 = RF_2$$
$$x_1 = -\frac{1}{R}x_2 \qquad (1)$$

The transformer ratio may be greater than or less than unity.

This relation can be written as a transfer matrix.

$$\begin{bmatrix} F_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & -R^{-1} \end{bmatrix} \begin{bmatrix} F_2 \\ x_2 \end{bmatrix} \qquad (2)$$

Since this is a static model, γ^2 is well defined and can be calculated.

$$\gamma^2 = \frac{-1}{T_{11}T_{22}} = 1 \qquad (3)$$

Returning to the actuator system model, it is assumed that the load and source have been characterized so that the load stiffness, $k_L(\omega)$, the load force spectrum, $|F_2(\omega)|^2$, and the source stiffness, $k_s(\omega)$, are known. The goal is to find an R such that the mean square force at the source is minimized, while delivering the desired force to the load. The mean square force can be evaluated in the frequency domain.

$$\langle F^2 \rangle = \int_0^\infty d\omega \, |F(\omega)|^2 \qquad (4)$$

Solving the mechanical circuit of Figure 1 gives the force required at the source.

$$F(\omega) = \left[\frac{1}{R}\frac{k_s(\omega)}{k_L(\omega)} + R\right] F_L(\omega) \tag{5}$$

The mean squared force, $< F^2 >$, is minimized by taking its derivative with respect to R and setting the resulting equation equal to zero.

$$\frac{\partial\langle F^2\rangle}{\partial R} = -\frac{1}{R^4}\int_0^\infty d\omega\ |F_L(\omega)|^2 \left|\frac{k_s(\omega)}{k_L(\omega)}\right|^2 + \int_0^\infty d\omega\ |F_L(\omega)|^2 = 0 \tag{6}$$

Solving for R gives the optimum transformer ratio.

$$R = \left[\frac{\int_0^\infty d\omega\ |F_L(\omega)|^2 \left|\frac{k_s(\omega)}{k_L(\omega)}\right|^2}{\int_0^\infty d\omega\ |F_L(\omega)|^2}\right]^{\frac{1}{4}} \tag{7}$$

This expression simplifies if k_s and k_L are real and independent of ω.

$$R = \sqrt{\frac{k_S}{k_L}} \tag{8}$$

This result can be understood as that ratio that scales the source stiffness so that it best approximates the stiffness of the load. For a static actuator system the stiffnesses are matched exactly. For dynamic system, the match is only the best possible since R is not a function of ω.

As a demonstration, for a static actuator system, the output impedance presented to the load can be evaluated.

$$k_2 = \frac{F_2}{x_2} = \frac{R^{-1}F_1}{R\,x_1} = R^{-2}k_L \tag{9}$$

Evaluating R^2 using equation 8.

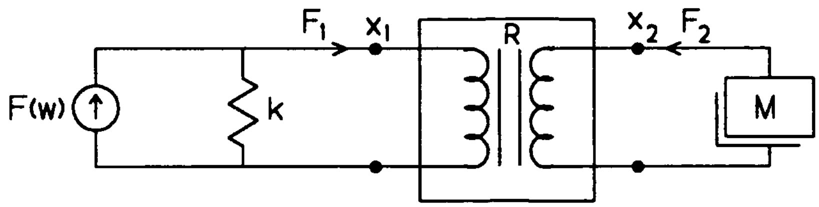
$$k_1 = k_s \tag{10}$$

Similarty, the actuator's output impedance presented to the load can be calculated.

$$k_2 = k_L \tag{11}$$

Previously it has been shown that in a static actuator system the maximum energy is delivered to the load when the actuator simultaneously matches load and source impedances. Thus maximizing force transfer, in the static limit, gives the same result as maximizing energy transfer.

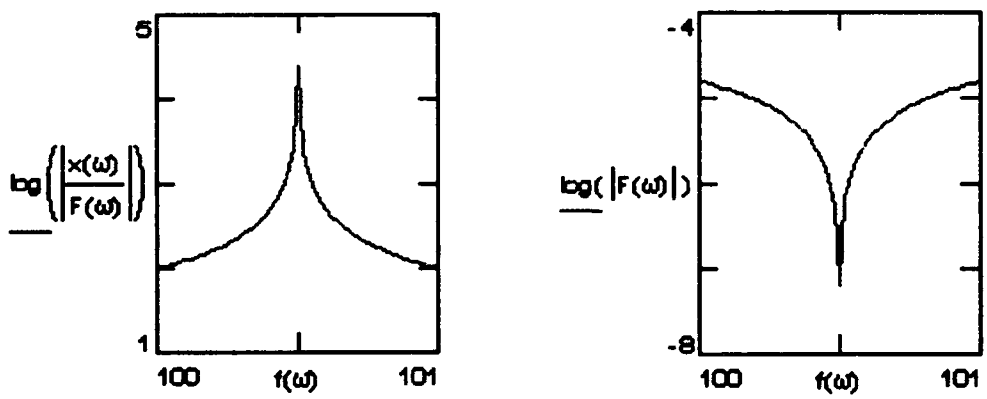
The following example illustrates the application of this optimization method, where the load is a mass and the source internal stiffness is a spring.



(F3)

Let M = 1 kg , k = 1 Newton/meter , and $F_2(\omega)$ = 1 Newton/[Hz]$^{1/2}$. The frequency range of interest will be 100 to 101 Hz, essentially a single frequency.

Using equation 7, the optimum transformer ratio is found to equal .00158. Using the optimum ratio and solving the mechanical circuit for the compliance seen by the source and for the required source power spectrum.

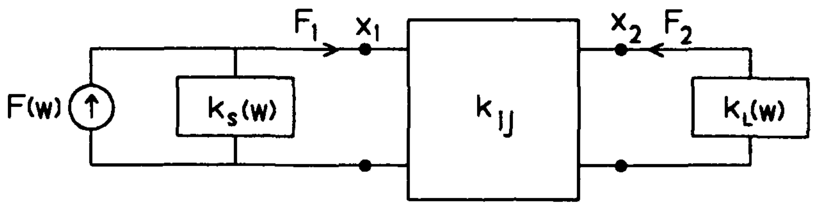


(F4)

It can be seen that the optimization method has sized the transformer to create a resonant system with the resonant frequency placed in the middle of the band. Given the task of designing a system for minimum force input one would most naturally create a resonant system. If the frequency band is broadened, the optimum transformer ratio still provides a resonant system but the resonance is shifted to the frequency requiring the most output of the source.

2.2 FREQUENCY INDEPENDENT ACTUATOR

The range of applications for an actuator system is significantly increased when the ideal transformer is replaced with a transduction element consisting of a frequency independent actuator.



(F5)

This model for the actuator system is quite useful because in many applications the actuator is well approximated as frequency independent.

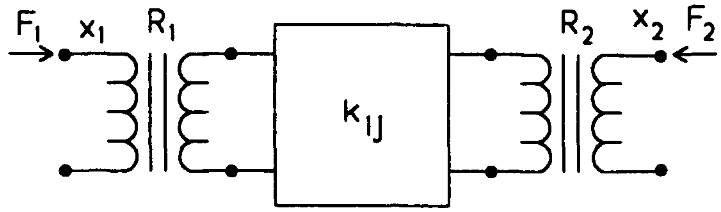
The properties of the actuator are described by a linear stiffness matrix.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{12}$$

As before, system optimization means minimizing the mean square force required of the source. This time the actuator matrix elements k_{ij} will be varied. However, one can not vary k_{11} and k_{22} arbitrarily. In varying the actuator elements γ^2 must be held constant otherwise the intrinsic properties of the actuator are changed. Variation of k_{ij}, while holding γ^2 constant, physically corresponds to changing the actuator's geometry, e.g., aspect ratio, number of wafers, turns ratio, etc.

For solid state actuators γ^2 is a true intrinsic variable. In the case of electrostrictive and magnetostrictive devices it can be expressed in terms of the transduction coefficient, Young's modulus, and the permittivity for electrostrictive materials or permeability for magnetostrictive materials. Non-solid state technologies do not have simple intrinsic material properties. The physical interpretation of γ^2 in these cases requires analysis specific to the individual technology.

To extend the meaning of intrinsic variables to non-solid state technologies consider the following. Add ideal transformers to the actuator's input and output terminals.



(F6)

The stiffness matrix of the combined three elements can be expressed in terms of the turn ratios, R_1 and R_2, and the stiffness matrix elements, k_{ij}.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} R_1^2 k_{11} & \frac{R_1}{R_2} k_{12} \\ \frac{R_1}{R_2} k_{12} & \frac{1}{R_2^2} k_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{13}$$

Calculating γ^2 for the overall stiffness matrix shows that the transformer ratios cancel leaving γ^2 unchanged.

$$\gamma^2 = \frac{k_{12}^2}{k_{11} k_{22}} \neq f\ (R_1, R_2) \tag{14}$$

This property can be used to generalize the concept of intrinsic variable to non-solid state actuators. An intrinsic variable will be defined as a variable which remains constant under this transformer scaling transformation.

Returning to optimization of the actuator system model, it is assumed that the load and source have been characterized by $k_L(\omega)$, $|F_2(\omega)|^2$, and $k_s(\omega)$. The goal is to find k_{11} and k_{22} such that the mean square force required at the source is minimized.

The mechanical circuit shown in Figure 5 can be solved for the force required of the source.

$$F(\omega) = \frac{1}{k_{12}} \left[k_{11} + k_s(\omega) + k_{22} \frac{k_s(\omega)}{k_L(\omega)} + \frac{k_{11} k_{22} (1 - \gamma^2)}{k_L(\omega)} \right] F_L(\omega) \tag{15}$$

The values of k_{11} and k_{22} which minimize $< F^2 >$ are found by substituting equation 15 into equation 4 and taking its derivative with respect to k_{11} and k_{22} while holding γ^2 constant. The resulting equations are set equal to zero and solved for k_{11} and k_{22}.

The solution for the optimum k_{11} is the real positive root of equation 16.

$$k_{11}^2 \int_0^\infty d\omega \; |F_2(\omega)|^2 \left|1 + \frac{\kappa(k_{11})}{k_L(\omega)} (1-\gamma^2)\right|^2 - \int_0^\infty d\omega \; |F_2(\omega)|^2 \; |k_s(\omega)|^2 \left|1 + \frac{\kappa(k_{11})}{k_L(\omega)}\right|^2 = 0 \tag{16}$$

Where the function $\kappa(k_{11})$ is defined in equation 17.

$$\kappa(k_{11}) = \sqrt{\frac{\int_0^\infty d\omega \; |F_2(\omega)|^2 \; |k_s(\omega) + k_{11}|^2}{\int_0^\infty d\omega \; |F_2(\omega)|^2 \; |k_L(\omega)|^{-2} \; |k_s(\omega) + k_{11}(1-\gamma^2)|^2}} \tag{17}$$

The stiffness matrix element, k_{22} is equal to κ evaluated using the solution for k_{11}.

$$k_{22} = \kappa(k_{11})$$

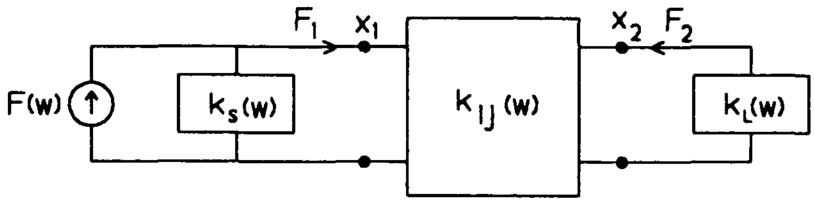
As a check, the static limit of the system model can be recovered by replace $k_L(\omega)$ and $k_s(\omega)$ with their static values. Equations 16 and 17 can be simplified.

$$k_{11} = \frac{k_s}{(1-\gamma^2)} \qquad k_{22} = \frac{k_L}{(1-\gamma^2)} \tag{18}$$

This is the result previously derived using the energy method.

2.3 FREQUENCY DEPENDENT ACTUATOR

A broader range of actuator systems are encompassed when all the elements of the system include frequency dependence.

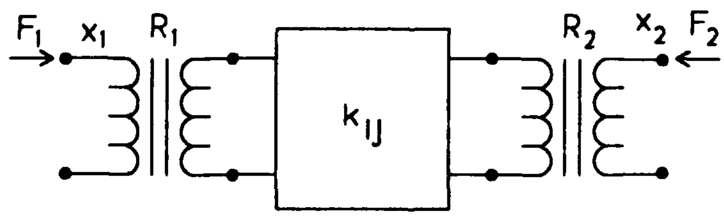


(F7)

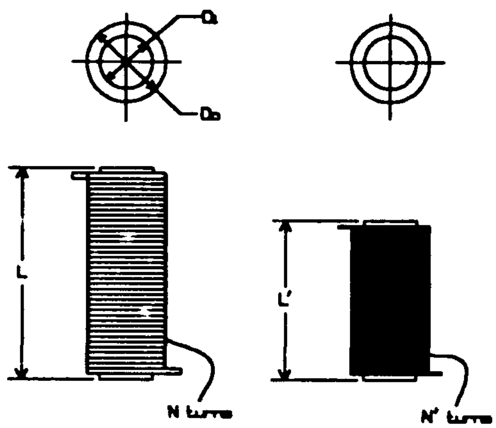
This system model, although quite powerful, is mathematically more difficult. It is recommended for use only when a frequency independent actuator model cannot be justified.

For a frequency dependent actuator, k_{11}, k_{22}, and γ^2 are no longer constants. Therefore, optimization can no longer involve their direct variation. A new approach is required. To be useful, the new approach should have the following properties: The parameters varied should be real and independent of frequency, since this is all that can be done and yield easily used results. The intrinsic properties of the actuator must not be affected. Optimization should result in solutions that reduce to those obtained in the static limit. Finally, changes in the parameters should correspond to changes in the design of the actuator, source, or load that make physical sense.

The parameters, R_1 and R_2, previously introduced in Figure 6 and reproduced below, have the desired properties. We are not implying that actual physical transformers be used, although this is certainly possible, but that for mathematical purposes one vary k_{11} and k_{22} through the parameters R_1 and R_2. An additional advantage of using R_1 and R_2 is that their scaling need not be applied to the actuator only. Their scaling effects may be partially or entirely applied to the source and/or load impedance. Finding the actuator design characteristics that can be varied in order to realize the scaling of R_1 and R_2 will differ for each actuator technology.



As an example of determining the design parameters to scale, consider a magnetostrictive actuator. The compliance matrix element, c_{11} can be expressed in terms of the actuator model shown in the figure below. The compliance matrix is the inverse of the stiffness matrix.



$$c_{11} = \frac{1}{R_1^2}\,\frac{1}{k_o} = \frac{1}{R_1^2}\,\frac{L}{E_o A}$$

$$R_1^2 = \frac{L}{L'} \tag{19}$$

We have assumed that the cross-sectional area is fixed and that only the actuator's length is scaled. The primed length is the new length.

Examine the electrical element of the compliance matrix.

$$c_{22} = R_2^2\,(L_f - j\,\frac{R}{\omega})$$

$$= R_2^2\,\frac{n^2}{L}\left[\mu_f A\left(\frac{d_o}{d_i}+1\right)^2 - j\,\frac{\pi}{\omega\,\sigma}\left(\frac{d_o+d_i}{d_o-d_i}\right)\right] \tag{20}$$

$$R_2^2 = \left(\frac{n'}{n}\right)^2 \frac{L}{L'}$$

The second scaling parameter is related to the number of turns of the coil surrounding the magnetostrictive material and its length. The cross-sectional area must remain fixed or the inductance and resistance will not scale the in the same way.

As a check, the scaling of the off diagonal stiffness matrix element can be calculated both from the model using equations 19 and 20, and from the scaling of the stiffness matrix using equation 13.

$$
\begin{aligned}
c_{12} &= \frac{-R_2}{R_1} n d_{33} \\
\frac{R_2}{R_1} &= \frac{n'}{n}
\end{aligned}
\qquad (21)
$$

The only parameter that can be scaled in the transduction element is the number of turns, thus the quotient of R_2 and R_1 must equal n' / n. This agrees with their quotient from equations 19 and 20.

Returning to the actuator system of Figure 7, it is again assumed that the load and source have been characterized so that $k_L(\omega)$, $|F_2(\omega)|^2$, and $k_s(\omega)$ are known. The goal is to find R_1 and R_2 such that the mean square force at the source is minimized. The multipliers have been placed within the actuator stiffness matrix following equation 13.

The mechanical circuit shown in Figure 7 can be solved for the force required of the source, $F(\omega)$.

$$
\begin{aligned}
F(\omega) = \frac{1}{k_{12}(\omega)} \Bigg[& R_1 R_2 k_{11}(\omega) + \frac{R_2}{R_1} k_s(\omega) + \frac{k_{22}(\omega)}{R_1 R_2} \frac{k_s(\omega)}{k_L(\omega)} \\
& + \frac{R_1}{R_2} \frac{k_{11}(\omega) k_{22}(\omega) (1 - \gamma(\omega)^2)}{k_L(\omega)} \Bigg] F_L(\omega)
\end{aligned}
\qquad (22)
$$

The values of R_1 and R_2 that minimize $< F^2 >$ are found by substituting equation 22 into equation 4 and taking its derivative with respect to R_1 and R_2, while holding k_{ij} constant. The optimum values are determined by setting the two resulting equations equal to aero and solving for R_1 and R_2.

The optimum R_1 is the real positive root of equation 23.

$$
\begin{aligned}
& R_1^4 \int_0^\infty d\omega \, |F_2(\omega)|^2 \left| k_{11}(\omega) \left(\sqrt{A(R_1)} + \sqrt{B(R_1)} \frac{k_{22}(\omega)}{k_L(\omega)} (1 - \gamma(\omega)^2) \right) \right|^2 \\
& - \int_0^\infty d\omega \, |F_2(\omega)|^2 \left| k_s(\omega) \left(\sqrt{A(R_1)} + \sqrt{B(R_1)} \frac{k_{22}(\omega)}{k_L(\omega)} \right) \right|^2 = 0
\end{aligned}
\qquad (23)
$$

In equation 23 the functions $A(R_1)$ and $B(R_2)$ are given in equation 24.

$$A(R_1) = \int_0^\infty d\omega \, |F_2(\omega)|^2 \left| \frac{k_{22}(\omega)}{k_L(\omega)} \left(k_s(\omega) + R_1^2 k_{11}(\omega) \, (1-\gamma(\omega)^2)\right) \right|^2$$
$$B(R_1) = \int_0^\infty d\omega \, |F_2(\omega)|^2 \, |k_s(\omega) + R_1^2 k_{11}(\omega)|^2 \tag{24}$$

The optimum value of R_2 can be found using equation 25.

$$R_2 = \left[\frac{A(R_1)}{B(R_1)}\right]^{\frac{1}{4}} \tag{25}$$

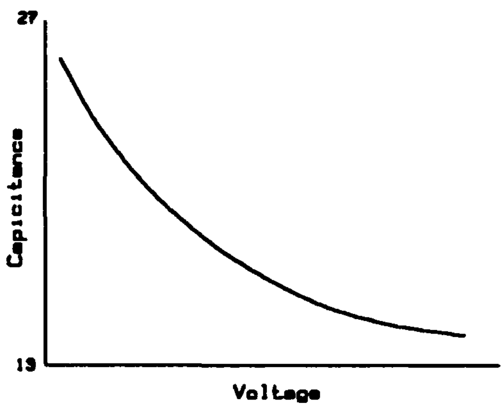
The static limit of this result can be verified by replacing $k_L(\omega)$, $k_s(\omega)$, $k_{11}(\omega)$, $k_{22}(\omega)$, and $\gamma^2(\omega)$ with constant values. Equations 24 and 25 then simplify to equation 18 which is the result previously derived using the energy method.

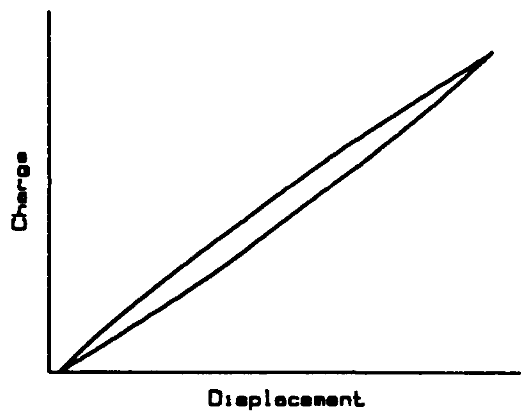
3.0 NON-LINEAR ACTUATOR SYSTEMS

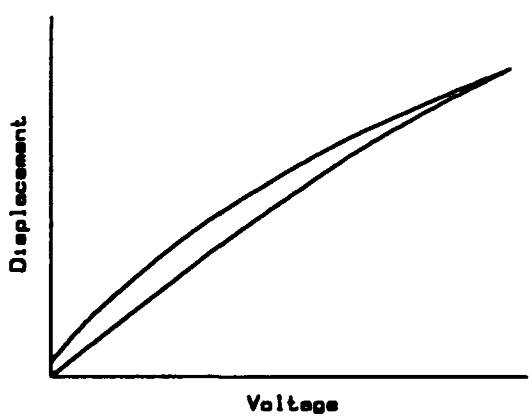
Nonlinearities occurring in an actuator system are primarily due to the power source and load. Examples are, voltage or current limits, pressure or flow limits, or displacement limits. These limits place restrictions on deliverable power. Nonlinearities occurring in the actuator itself are usually small compared to those in the power source or load. Since the source and load are the dominate nonlinear elements, simplified nonlinear models for both are developed and then used to estimate deliverable power. A nonlinear model for the actuator is also developed, but its use is limited since solutions must be performed in the time domain.

3.1 ACTUATOR MODEL

An example of the nonlinearities that occur in actuators is a piezoelectric formulation of lead magnesium niobate (PMN). The following plots are data showing the variation of the physical parameters for a commercially available sample of PMN. This variability will be reflected in the stiffness matrix elements which will show a similar degree of variability.

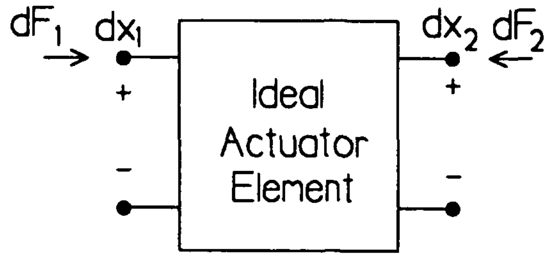






The matrix elements are nearly linear over the range of operation and for modeling purposes can be assumed to be linear. If necessary, nonlinear gain can be used in the power source to reduce the distortion due to weak nonlinearities.

Actuator nonlinearities can be modeled by modifying the formalism that has been developed for linear actuators. The modification consists of using the stiffness matrix of the general linear element with the input and output variables replaced by differentials.



(F9)

The model is then a nonlinear stiffness matrix with time independent elements.

$$\begin{bmatrix} dF_1 \\ dF_2 \end{bmatrix} = \begin{bmatrix} k_{11}(x_1, x_2) & k_{12}(x_1, x_2) \\ k_{21}(x_1, x_2) & k_{22}(x_1, x_2) \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} \qquad (26)$$

The matrix elements are assumed to be single valued functions, i.e., no hysteresis, of the independent variables x_1 and x_2. Using this model requires solving coupled nonlinear differential equations.

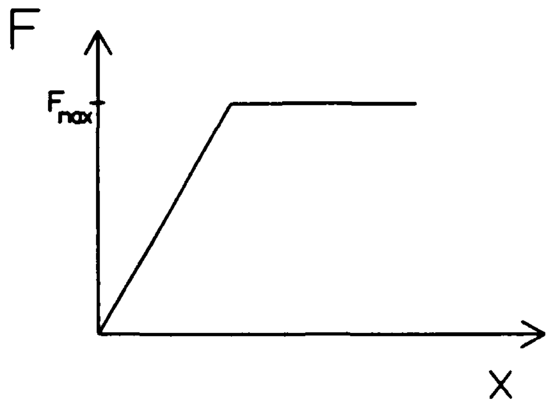
Unfortunately, this differential formulation does not lend itself to frequency domain analysis. Solutions must be performed in the time domain. The complexity and specificity of time domain analysis suggests that the general analysis of actuator nonlinearities is best suited to detailed design. For the purposes of technology selection, the use of a linear or quasi-linear actuator model is sufficient.

3.2 SOURCE AND LOAD MODELS

Our primary concern in this report is technology selection for power critical systems. Given this objective, the primary nonlinear effect limiting power output is saturation, i.e., clipping. Saturation limits the maximum work an actuator system can deliver to a load. If a specific amount of work must be delivered, saturation will require that the physical size and power consumption of the actuator system be increased.

Saturation levels usually occur in the source and are set by physical failure limits in the actuator such as voltage breakdown, heating due to power dissipation, or seal rupture pressure. Prudent design will limit the power source's output so that damage to the actuator is prevented. Thus, modelling saturation as occurring in the source is most natural. Load saturation can be important in low impedance systems. In low impedance systems stops may limit displacement.

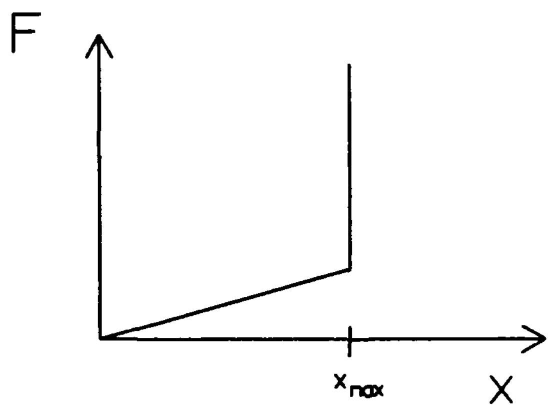
Since linear systems are the only ones that can be easily analyzed in detail, a piecewise linear model will be adopted for source saturation.



(F10)

The saturation level F_{max} is a generalized variable and can take the form of voltage, current, pressure, etc.

Similarly a piecewise linear model is adopted for the load.



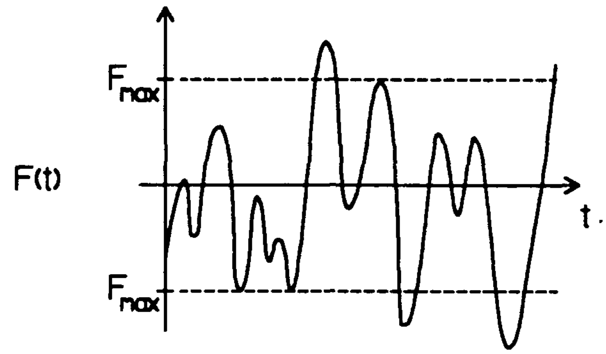
(F11)

3.3 SYSTEM POWER OUTPUT ESTIMATION

As previously indicated, maximum output of an actuator system is limited by saturation. If driven beyond saturation the source will clip and introduce distortion at the load. The maximum output of an actuator system is limited by the acceptable level of distortion. Distortion can be reduced by increasing the size of the actuator system. A tradeoff can be made between an acceptable level of distortion and system size. For a given level of distortion, comparisons between competing actuator technologies can be used for technology selection.

Source saturation occurs much more frequently that load saturation. The piecewise linear source model will used to estimate the impact of source clipping on power output. Similar analysis can be carried out for load saturation.

Total distortion due to source saturation will be estimated by the power unavailable to the load due to clipping. We will assume that the required output of the source can be approximated as a Gaussian random process.



(F12)

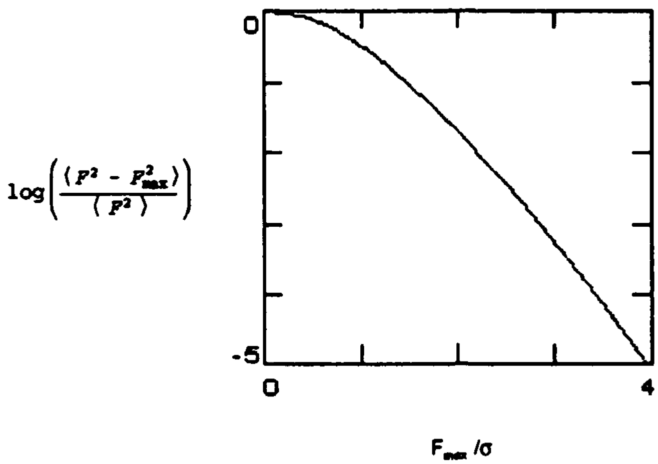
The measure of unavailable power is the mean squared force required of the source that exceeds the source's saturation level.

$$\langle F^2 - F_{max}^2 \rangle = \frac{1}{\sigma\sqrt{2\pi}} \int_{F_{max}}^{\infty} dF \, (F^2 - F_{max}^2) \, e^{-\left(\frac{F^2}{2\sigma^2}\right)}$$

$$= F_{max}^2 \left[\left[\left(\frac{F_{max}}{\sigma} \right)^{-2} - 1 \right] erfc\left(\frac{F_{max}}{\sigma\sqrt{2}} \right) + \sqrt{\frac{2}{\pi}} \left(\frac{F_{max}}{\sigma} \right)^{-1} e^{-\frac{1}{2}\left(\frac{F_{max}}{\sigma}\right)^2} \right] \quad (27)$$

where erfc(x) is the complimentary error function.

To apply this method the required output of the source must be statistically characterized by obtaining its standard deviation, σ. The acceptable level of distortion is used to select the source saturation level. This sizes the actuator and the source itself. For example, setting F_{max} at 3σ results in a distortion of .05% or -33 dB. The effect of varying the saturation level is shown below.



(F13)

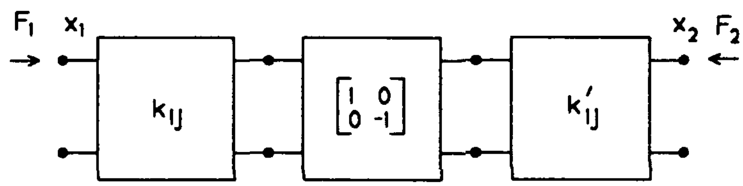
4.0 COMPOUND ACTUATORS

Often a designer will want to connect actuators of differing technologies to produce a compound actuator. An example is an electrostrictive actuator attached in series with a hydraulically operated piston. The piston would provide large scale slow response displacement while the electrostrictive actuator provides small high response motion.

Transformers, which are a special case of an actuator, can also be connected in this way. Rather than adjusting an actuator's geometry to provide an impedance match with a source or load, a physical transformer can be used. The transformer may be either electrical or mechanical in nature.

4.1 LINEAR MODELS

Compound actuators are formed by cascading individual actuators together. The new actuator formed by the combination has a stiffness matrix with an "efficiency" measure, $\tilde{\gamma}^2$.



(F14)

The resulting compound stiffness matrix is determined from the product of the individual transfer matrices. The diagonal matrix between the stiffness matrices is required to maintain the correct sign convention between the output variables of k_{ij} and the input variables of k'_{ij}

The compound transfer matrix is determined by multiplying the transfer matrices according to Figure 14.

$$\begin{bmatrix} F_1 \\ x_1 \end{bmatrix} = \frac{1}{k_{12}k'_{12}} \begin{bmatrix} [k_{11}k'_{11} + k_{11}k_{22}(1-\gamma^2)] & -[k_{11}k'_{11}k'_{22}(1-\gamma'^2) + k_{11}k_{22}k'_{22}(1-\gamma^2)] \\ [k'_{11} + k_{22}] & -[k_{22}k'_{22} + k'_{11}k'_{22}(1-\gamma'^2)] \end{bmatrix} \begin{bmatrix} F_2 \\ x_2 \end{bmatrix} \tag{28}$$

This transfer matrix can be rearranged in the form of a stiffness matrix.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{1}{k'_{11} + k_{22}} \begin{bmatrix} k_{11}k'_{11} + k_{11}k_{22}(1-\gamma^2) & k_{12}k'_{12} \\ k_{12}k'_{12} & k_{22}k'_{22} + k'_{11}k'_{22}(1-\gamma'^2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{29}$$

The resulting $\tilde{\gamma}^2$ for the compound actuator will be less than the product of the individual γ^2.

$$\tilde{\gamma}^2 = \frac{\gamma^2 \gamma'^2}{1 + (1-\gamma^2)(1-\gamma'^2) + \frac{k_{22}}{k'_{11}}(1-\gamma^2) + \frac{k'_{11}}{k_{22}}(1-\gamma'^2)} \qquad (30)$$

4.2 NON-LINEAR MODELS

Non-linear compound actuators are cascaded in a manner identical to that previously described. The resulting stiffness matrices will consist of nonlinear differential equations with nonconstant coefficients. Solutions to these equations must be performed in the time domain. This becomes a detailed design technique and is not particularly useful for actuator technology selection.

5.0 SYSTEM TECHNOLOGY SELECTION MEASURES

An actuator optimization formalism has been developed which combines all the developments presented up to this point. This formalism will enable designers to select the best technology for an application. Our emphasis is on actuator systems for power critical applications, as these are of great interest and their design typically has not been fully optimized. The formalism provides measures which allow simple and rapid comparisons of candidate actuator systems across a wide range of technologies. The formalism can also be used for detailed design of actuator systems, but this is not our primary goal at this time.

An optimal actuator system will not only match the best actuator technology to the load, but also use the best power source technology for the actuator. For example, an electrostrictive actuator is most efficient when driven by a source those output impedance is capacitive. Similarly, a magnetostrictive actuator is most efficient when coupled to an inductive power source. Thus, an optimal actuator system must simultaneously match the load, source, and actuator. At the time, most manufacturers of power sources do not specify detailed output impedance information. The characterization of power sources is a topic that will be addressed in the next interim report.

5.1 SYSTEM CHARACTERIZATION

It is assumed that the actuator system designer wishes to select the best actuator and source technology combinations for the application in the most efficient way. Consequently, the selection process begins by deciding which of the following three categories best describes the application.

Static actuator - Static load and source
Static actuator - Dynamic load and source
Dynamic actuator - Dynamic load and source

As it has already been shown, analysis becomes much more complicated with the addition of dynamics to the models. Therefore, the simplest suitable model should be used. Dynamic here implies frequency dependency.

After deciding the models to be used, the designer will need information about the actuator and source technologies under consideration as well as the load characteristics. The load must be characterized in terms of its impedance, the required input force spectrum, and the level of distortion that can be tolerated. For the actuators and sources, stiffness matrices and output impedances are necessary. An actuator's stiffness matrix is a strong function of its geometry. Physical design considerations will generally place limits on actuator geometry. For selection purposes, the initial actuator geometry should resemble its desired final geometry.

With all the necessary information in hand the optimization formalism can be applied. The route to finding the best technology for the application will be different depending on the modeling category selected. Therefore, each category is presented separately.

5.2 STATIC ACTUATOR - STATIC LOAD/SOURCE

One usually does not know before hand the actuator technology that will best match a given source. During the selection process a number of actuator technologies must be combined with sources for evaluation.

Evaluation of each actuator/source combination proceeds by finding the optimum k_{11} and k_{22} from equation 18. Candidate combinations are then eliminated based on the ratio between the initial k_{11} and k_{22} and the optimum $k_{11\,opt}$ and $k_{22\,opt}$.

$$R_{1\,opt}^{2} = \frac{k_{11\,opt}}{k_{11}} \qquad\qquad R_{2\,opt}^{2} = \frac{k_{22}}{k_{22\,opt}}$$

Experience has shown that values of $R_{1\,opt}$ or $R_{2\,opt}$ greater than 10 or less than 0.1 correspond to scalings of the design that are usually impractical due to poor aspect ratios, unreasonable number of wafers or turns, etc. This is a guide however and may be modified as needed. For example, if one did a particularly poor job of initially designing the actuator, then an $R_{1\,opt}$ or $R_{2\,opt}$ outside of the recommended range can correspond to an actuator geometry that is more practical. Obviously, candidate combinations with $R_{1\,opt}$ and $R_{2\,opt}$ close to one indicate a natural match between actuator, load, and source.

Next the total input energy required for each of the remaining combinations are calculated and compared, using $R_{1\,opt}$ and $R_{2\,opt}$. The combination requiring the least energy input to the source will be the optimum combination of technologies for the application.

Each source will require a specific input power to drive the source to saturation, often available from the manufacture's literature. The actuator/source combination with the lowest required input power will be the optimum combination of technologies for the application. Also, other considerations such as cost and reliability may be used to select an actuator that may have a similar efficiency but other features that are desirable. One advantage of the static model is that energy is used as the comparison basis and it can be used universally across all technologies. This is not the case with the dynamic models.

5.3 STATIC ACTUATOR - DYNAMIC LOAD/SOURCE

Evaluation of each actuator/source combination proceeds by finding the optimum k_{11} and k_{22} from equations 16 and 17. Candidate combinations are then eliminated based on the ratio between the initial k_{11} and k_{22} and the optimum $k_{11\,opt}$ and $k_{22\,opt}$.

$$R_{1\,opt}^{2} = \frac{k_{11\,opt}}{k_{11}} \qquad\qquad R_{2\,opt}^{2} = \frac{k_{22}}{k_{22\,opt}}$$

Again, values of $R_{1\,opt}$ or $R_{2\,opt}$ greater than 10 or less than 0.1 is usually impracticable due to poor aspect ratios, unreasonable number of wafers or turns, etc.

Next for the actuator/source combinations that remain, the minimum mean square generalized force is calculated from equations 4 and 15 using $R_{1\,opt}$ and $R_{2\,opt}$. Depending on the actuator/source combination, the mean square force will have units of volts^2, amps^2, or psi^2. Comparisons across actuator technologies cannot be made at this point since it is not possible to equate mean square force with differing units. Where actuator technologies are comparable, the minimum mean squared input force are compared and within each group the actuator with the smallest value selected.

The final selection for the best actuator/source combination is based on total power input. For each remaining combination the distortion requirement is applied and the saturation level for each power source determined. Each source will require a specific input power to drive the source to saturation, often available from the manufacture's literature. The actuator/source combination with the lowest required input power will be the optimum combination of technologies for the application. Also, other consideration such as cost and reliability can be used at this point.

5.4 DYNAMIC ACTUATOR - DYNAMIC LOAD/SOURCE

The selection process is very similar to that of the static actuator - dynamic load/source model.

Evaluation of each actuator/source combination proceeds by finding the optimum $R_{1\,opt}$ and $R_{2\,opt}$ from equations 23, 24, and 25. Again, values of $R_{1\,opt}$ or $R_{2\,opt}$ greater than 10 or less than 0.1 are usually impracticable due to poor aspect ratios, unreasonable number of wafers or turns, etc.

Next, for the actuator/source combinations that remain, the mean square generalized force is calculated from equation 22 using $R_{1\,opt}$ and $R_{2\,opt}$. Depending on the actuator/source combination, the mean square force will have units of volts^2, amps^2, or psi^2. Comparisons across actuator technologies cannot be made at this point since it is not possible to equate mean square force with differing units. Where actuator technologies are comparable, the

minimum mean squared input force are compared and within each group the actuator with the smallest value selected.

The final selection for the best actuator/source combination is based on total power input. For each remaining combination the distortion requirement is applied and the saturation level for each power source determined. Each source will require a specific input power to drive the source to saturation, often available from the manufacture's literature. The actuator/source combination with the lowest required input power will be the optimum combination of technologies for the application. Also, other consideration such as cost and reliability can be used at this point.

6.0 SUMMARY

A procedure for evaluating and selecting power actuator systems has been developed. This procedure uses the power source impedance, load impedance, spectrum of the required load force, and signal distortion requirement to find measures that can be used to select the best actuator technology for power critical applications. The procedure is applicable to a wide range of technologies. The only restrictions are that the actuator technology be representable by a generalized stiffness matrix.

One outcome of the work reported here is the realization that power source design is a very critical issue for actuator systems. There are two areas that need work; power source characterization and design of optimal power sources. In the next period we will concentrate our efforts in these areas.